measured with the plane of the reference at the short circuit is shown in Fig. 9(a). The measurement of the impedance locus of a short circuit referenced so as to give one complete circle on the reflection coefficient plane as the frequency is varied over the 12 per cent X-band is shown in Fig. 9(b). The display on the expanded reflection coefficient plane of the impedance locus of the single tuned circuit described above is shown in Fig. 10(a). Comparison of these photographs with the results yielded by the waveguide circuit alone (Figs. 6, 7, and 8) indicate that the accuracy of the entire system is essentially the same as the accuracy of the displays plotted from the output voltages of the waveguide circuit.

Finally, to further illustrate the usefulness of the automatic impedance plotter, a photograph of the expanded display resulting from the impedance measurement of a multiple-tuned waveguide filter circuit having a complex impedance locus is shown in Fig. 10(b). A measurement of this impedance locus by the usual slotted line techniques takes at least one hour; the measurement is performed instantaneously by the automatic impedance plotter following a setup time of about ten minutes. In addition, the measurement by the impedance plotter insures that all of the details of the impedance locus will be observed since the measurement is performed continuously over the frequency band.

It should be noted that in the photographs of the impedance plotter display, the crt trace at the center of each photo indicates the center of the reflection coefficient plane ($w=0$). The crt trace is returned to the center of the screen once during each frequency sweep cycle so as to indicate the impedance locus for the match condition.

A Method of Measuring Dissipative Four-Poles Based on a Modified Wheeler Network*

H. M. ALTSCHULER†

Summary—**A method of abstracting the parameters of dissipative four-poles from measured data is presented here. This semi-precision method is applicable to symmetric four-poles and results directly in a conveniently symmetric network representation. It is based on the modified Wheeler representation, a new and completely general network, which is introduced in this paper. In addition to the derivation of the network and of the analysis, the relationship that the network bears to its dual, to the impedance Tee, and to the admittance Ri is presented.**

Introduction

THE experimental procedures in four-pole measurements and the subsequent analyses of data can be divided roughly into two classes: "point" measurements on the one hand, and "precision" and "semi-precision" measurements on the other. While point measurements require that a specific and minimum number of datum points be taken and analyzed, precision and semi-precision measurements require that a sufficiently large number of different datum points are taken to be analyzed later by a technique which effectively averages them. In the latter class of measurements, which is of interest here, essentially two methods are available for measuring dissipative four-poles, as far as the author knows: the method due to Felsen and Oliner[1] and its variations, and Deschamps' method.[2]

It is pointed to note that, while the Felsen-Oliner networks are constituted of conventional network elements, they are not capable of displaying structural symmetry explicitly. The Deschamps method, in contrast, results in a scattering representation, and as such displays the symmetry (if any) of the measured structure explicitly, i.e., for symmetric structures, regardless of reference planes employed in the representation, $|S_{11}| = |S_{22}|$. It is apparent, then, that no precision or semi-precision measurement method has been available which *directly* results in a network (as opposed to a matrix representation) capable of representing symmetric structures in a symmetric fashion. To obtain such a network from precision or semi-precision measurements, it has been necessary to transform either the scattering representation or one of the Felsen-Oliner net-

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[1] L. B. Felsen and A. A. Oliner, "The Precision Measurement of Equivalent Circuit Parameters of Dissipative Microwave Structures," Report R-282-52, PIB-221, Polytechnic Inst. of B'klyn, Microwave Res. Inst.; November, 1952; also "Determination of equivalent circuit parameters for dissipative microwave structures," Proc. IRE, vol. 42, pp. 477–483; February, 1954.

[2] G. A. Deschamps, "Determination of reflection coefficients and insertion loss of a waveguide junction," *Jour. Appl. Phys.*, vol. 24, pp. 1046–1050; August, 1953.

works. Although such transformations are conceptionally simple, the computations involved are time consuming. In addition to its simplicity and to the physical insight which a network of symmetric form may provide, it also permits the application of a bisection theorem. In contrast, neither the Felsen-Oliner networks nor the scattering representation resulting from the Deschamps method can be bisected in the usual simple fashion.

In this connection the work of Harold A. Wheeler[3,4] has been highly suggestive in that it describes a break up of four-poles into two reactive portions separated by a resistive portion. It is this form which has been used as a starting point here for the development of a general network representation which is symmetric in appearance (and can hence be bisected) and which results naturally from the analysis of measured data. Although Wheeler describes neither the network given here nor the method of measuring it, both its form and its measurement are to some extent implicit in his work. It has consequently been named "modified Wheeler" network.

The modified Wheeler network can be employed only in the dissipative case, although formally it is also a proper representation for a lossless four-pole in the limit as the no-loss case is approached in some systematic fashion. Although the network itself is of interest as a representation apart from measurements, the semi-precision method of measuring symmetric four-poles based on it is stressed here and is recommended. The precision method of measuring general four-poles is comparable to the precision method due to Felsen and Oliner except that the resulting network, of course, is different.

Development of the Modified Wheeler Network

In general, according to Wheeler,[4] any four-pole may be represented by a simple attenuator located between two purely reactive networks. Since this network form can include more than six circuit parameters necessary to specify a four-pole, various assumptions may be made regarding the excess parameters. In the present case, and without loss of generality, it is assumed that the attenuator is symmetric and that it is matched in both directions to the characteristic impedance of the input transmission line of the structure being represented. The attenuator can therefore be specified by a single number $|\Gamma_\alpha|$, the meaning of which will be made clear later. Fig. 1 shows this more restricted form, which constitutes the basis of the work presented. Here β and β' are the phase constants, and unity denotes the normalized characteristic impedance of the adjoining transmission lines.

[3] Harold A. Wheeler and David Dettinger, "Measuring the Efficiency of a Superheterodyne Converter by the Input Impedance Circle Diagram," Wheeler Monograph No. 9, March, 1949.

[4] Harold A. Wheeler, "The Transmission Efficiency of Linear Networks and Frequency Changers," Wheeler Monograph No. 10, May, 1949.

For the moment, let the attenuator be represented by a symmetric Tee with series arms R_1 and shunt arm R_2, which are related by

$$R_1 = \sqrt{R_2^2 + 1} - R_2, \tag{1}$$

since the attenuator is taken to be symmetric and matched. When the attenuator is terminated in any impedance Z_2, its input impedance Z_1 is then obtained as

$$Z_1 = \frac{1 + Z_2\sqrt{R_2^2 + 1}}{\sqrt{R_2^2 + 1} + Z_2}, \tag{2}$$

in view of (1). It follows that the input reflection coefficient of the attenuator has the value

$$\Gamma_1 = \frac{\sqrt{R_2^2 + 1} - 1}{\sqrt{R_2^2 + 1} + 1} \cdot \frac{Z_2 - 1}{Z_2 + 1} \equiv |\Gamma_\alpha| \cdot \Gamma_2. \tag{3}$$

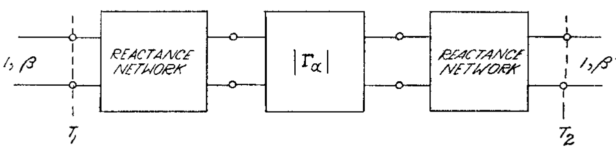

Fig 1—A general network consisting of symmetric matched attenuator between reactive four-poles.

Both factors on the right-hand side of this equation are in the form of the reflection coefficient. The first, which is seen to be positive real, is associated only with the attenuator, while the second is simply the reflection factor of the load impedance. The symbol $|\Gamma_\alpha|$ has consequently been adopted to denote a real positive reflection coefficient, specifically the reflection coefficient of the open-circuited attenuator. In view of its simple operational behavior, $|\Gamma_\alpha|$ will be represented here by a special network symbol as shown in Fig. 2(a).

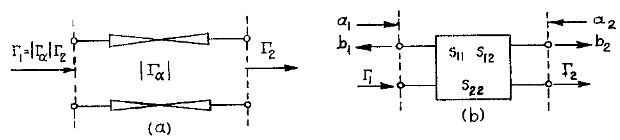

Fig. 2—(a) Reflection coefficient transformer; (b) Scattering four-pole.

Since the attenuator element is matched in both directions, the scattering coefficients S_{11} and S_{22} both equal zero. S_{12}, on the other hand, takes on the value $\sqrt{|\Gamma_\alpha|}$. In accord with the definitions of the incident and reflected quantities (see Fig. 2(b)) one writes the usual scattering equations

$$\begin{aligned} b_1 &= a_1 S_{11} + a_2 S_{12}, \\ b_2 &= a_1 S_{12} + a_2 S_{22}, \\ \Gamma_1 &= b_1/a_1, \qquad \Gamma_2 = a_2/b_2, \end{aligned} \tag{4}$$

from which it is recognized at once that, for the element of Fig. 2(a),

$$b_1 = a_2\sqrt{|\Gamma_\alpha|}\ ;\ a_1 = b_2/\sqrt{|\Gamma_\alpha|}\ ;\ \Gamma_1 = |\Gamma_\alpha|\,\Gamma_2. \qquad (5)$$

This is entirely analogous to the equations

$$E_1 = E_2\sqrt{N},\quad I_1 = I_2/\sqrt{N},\quad Z_1 = NZ_2, \qquad (6)$$

where the subscripts 1 and 2 again refer to input and output; E, I and Z are voltage, current and impedance, and N is the impedance ratio of a transformer. It is therefore seen that the operational character of $|\Gamma_\alpha|$ corresponds exactly to that of N. To underline this "transformer" behavior, the attenuator element will consequently be referred to as a "reflection coefficient transformer."

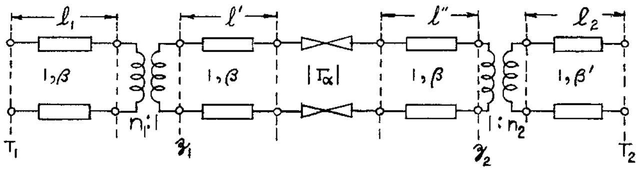

Fig. 3—Reflection coefficient transformer between lossless four-poles.

It is well known that a network consisting of two lossless transmission lines coupled by a transformer is a general representation of a reactive four-pole.[5] In view of this fact (and of the reflection coefficient transformer defined) the representation shown in Fig. 3 is entirely equivalent to that of Fig. 1. In the network of Fig. 3, the order of the three elements l', $|\Gamma_\alpha|$ and l'' may be interchanged arbitrarily, since each of these elements operates on the reflection coefficient in a multiplicative fashion, i.e.,

$$\Gamma(z_1) = (e^{-j2\beta l'}\,|\Gamma_\alpha|\,e^{-j2\beta l''})\Gamma(z_2). \qquad (7)$$

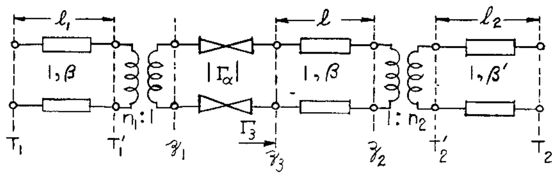

Fig. 4—Modified Wheeler representation.

The two transmission lines can consequently be grouped together as a single line of length l, where $l = l' + l''$. The resulting network is that shown in Fig. 4. It is this last representation[6] which has been named the "modified Wheeler" network. In the network of Fig. 4, the reflection coefficient transformer and the transmission line l may be incorporated to form a single lossy transmission line with attenuation constant α, so that

$$\Gamma(z_1) = e^{-2(\alpha+j\beta)l}\Gamma(z_2);\qquad \alpha \equiv \frac{1}{2l}\ln\frac{1}{|\Gamma_\alpha|}\cdot \qquad (8)$$

On employing this notation the four-pole may be pictured in the form shown in Fig. 5. Here the transmission line is somewhat peculiar in that it has the real characteristic impedance of the input transmission line and a complex propagation constant. This situation, however, is well approximated in many practical lossy transmission lines. It is seen that the *form* of the last representation is perfectly symmetric so that it becomes particularly suitable for representing symmetric structures. Of course the network of Fig. 4 also has a symmetric form, although it appears to be asymmetric to the eye. One can, for instance, picture it in the form of Fig. 3 with $l' = l'' = l/2$. While Figs. 4 and 5 represent arbitrary four-poles at any set of reference planes, it is clear that one may represent any microwave structure at the special planes T_1' and T_2' by only the four parameters n_1, $|\Gamma_\alpha|$, l and n_2 (or n_1, α, l, n_2). Such a reference plane shift leaves the remaining parameters conveniently unaltered.

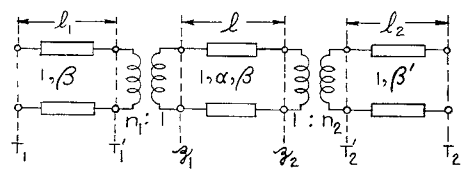

Fig. 5—Modified Wheeler representation employing a lossy transmission line.

Modification of the Reflection Coefficient Locus by the Various Circuit Elements

The representation which formed the point of departure is a perfectly general representation of a (linear, passive, reciprocal) four-pole. It follows that the network of Fig. 5 (or 6, facing page) is general. However, for the sake of completeness and since it is instructive to follow the step-by-step transformation of the reflection coefficient locus by the various network elements, an independent proof of generality is given below.

Let it be assumed that the various values of Γ_i, the input reflection coefficient of a four-pole, corresponding to various reactive output terminations Γ_0 are known. The output reflection coefficient locus is, of course, the unit circle, while the corresponding input locus is some circle as shown in Fig. 6(a). The center of the Γ_i circle, which is located on a radial line $\theta = \theta_0$ can be shifted to the $\theta_1 = 0$ axis in the Γ_1-plane when Γ_i and Γ_1 are related by a transmission line l_1 as shown in Fig. 6(b). Likewise, the circle with its center on the real axis of the Γ_1 plane can always be transformed into a circle with center at the origin at the Γ_2 plane when Γ_1 and Γ_2 are related by a suitably chosen transformer as shown in Fig. 6(c). Whereas the radius of the circle does not change as a result of the transformation from Γ_i to Γ_1, it does change as a result of the step from Γ_1 to Γ_2. When Γ_2 and Γ_3 are

[5] A. Weissfloch, "Eine Transformation über Verlustlose Vierpole und seine Anwendung," *Hochfrequentz und Electroakustik*, vol. 60, pp. 67–73.

[6] Since the completion of the work presented here the author has become aware of the fact that the same representation has been employed independently by Georges A. Deschamps, of Federal Telecommunication Laboratories, in connection with as yet unpublished material.

related by the reflection coefficient transformer of the proper value, the radius is increased to become unity in the Γ_3 plane, as shown in Fig. 6(d). Hence Γ_3 represents the input locus of a lossless four-pole. As already pointed out, such a lossless four-pole can always be represented by two transmission lines separated by a transformer element. The transformation effected by the lossless four-pole does not, of course, alter the locus in question, but only redistributes the points on it.

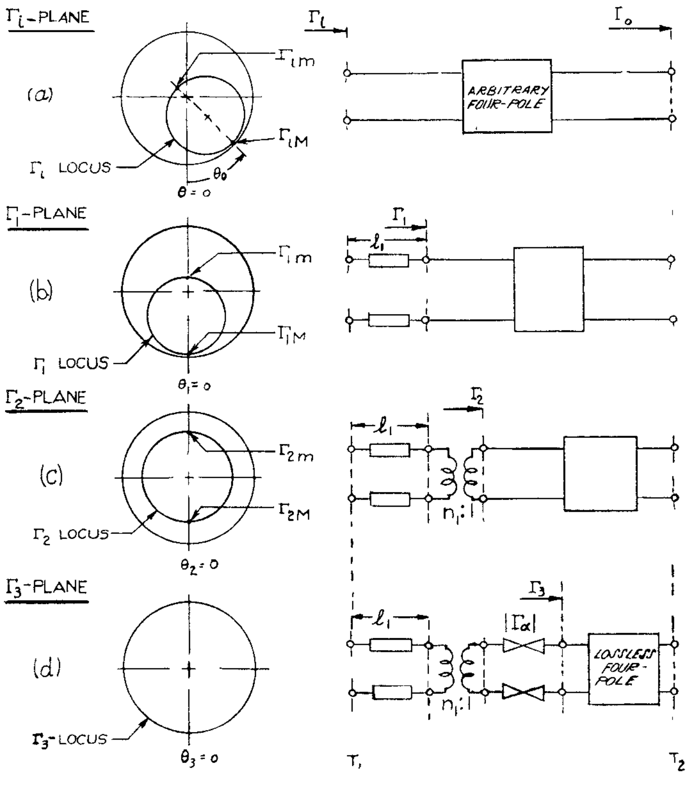

Fig. 6—Modification of the reflection coefficient locus by various network elements.

The foregoing steps show that for any set of input reflection coefficients corresponding to specific reactive output terminations there exists a modified Wheeler representation. This is equivalent to the assertion that this network is capable of representing any (passive, linear, reciprocal) four-pole.

When the four-pole in question is perfectly lossless the reflection coefficient transformer vanishes, i.e., $|\Gamma_\alpha| = 1$. Under these conditions the parameters l_1, l_2, l, n_1 and n_2 still remain as part of the representation and may be used in a relatively arbitrary fashion to characterize the structure. If, in contrast, the condition of no loss in the four-pole is approached as a limit in a specific manner, i.e., when $|\Gamma_{im}|$ is defined as a function of $|\Gamma_{iM}|$ such that both approach unity simultaneously, but in different fashions, the parameters in question are well defined. Regardless of what this function is, $|\Gamma_\alpha|$ approaches unity in the limit. Here, as is implied by Fig. 6, Γ_{iM} and Γ_{im} are defined as the reflection coefficients (located on the circle) in the Γ_i plane which have the maximum and minimum absolute values respectively. Γ_{1M}, Γ_{1m}, Γ_{2M} and Γ_{2m} are their transforms in the Γ_1 and the Γ_2 planes.

Derivation of a Semi-Precision Measurement Procedure for Symmetric Four-Poles

As mentioned earlier, a procedure for obtaining a symmetric network representation for symmetric structures is of particular interest. The modified Wheeler network may form the basis either for a precision measurement of arbitrary four-poles or for a semi-precision measurement of symmetric four-poles, i.e., four-poles which are known to be symmetric with respect to some plane. The latter measurement is considered below.

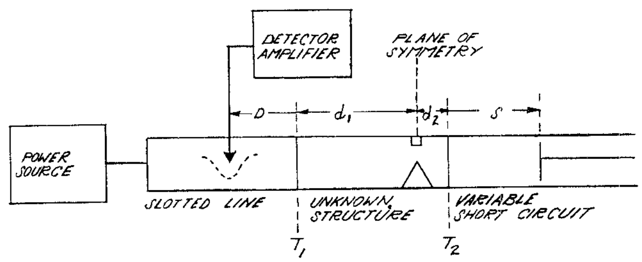

Fig. 7—Outline of experimental apparatus.

Let it be assumed that a structure known to be symmetric with respect to some plane is measured with the apparatus outlined in Fig. 7. In this measurement the planes T_1 and T_2 (i.e., the ends of the structure to be measured) need not be symmetric with respect to the plane of symmetry. D and S are the distances from T_1 and T_2 to the probe (located at the voltage minimum) and to the short circuit, respectively. Corresponding to a series of settings of the short circuit, S, values of D, and vswr are measured and input reflection coefficients are computed via the standard equation

$$\Gamma_i = \frac{\text{vswr} - 1}{\text{vswr} + 1} e^{j(2\beta D + \pi)} \equiv |\Gamma_i| e^{j\theta}. \tag{9}$$

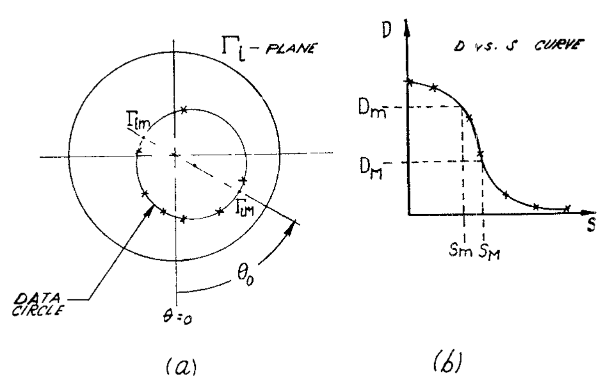

Fig. 8—Data plots necessary for semi-precision analysis of symmetric structure.

The computed values are then plotted in polar coordinates. The best circle is fitted through the data as shown in Fig. 8(a) and a D vs. S curve, such as that shown in Fig. 8(b), is also drawn.

From this plotted data the various circuit elements are then abstracted as is shown below. Initially, the steps outlined in connection with Fig. 6 will be followed. Since the center of the circle in the Γ_1 plane has been required to lie on the positive real axis, while its angular location in the Γ_i plane is θ_0, it can be seen that l_1, the length of the transmission line connecting these planes, is given by

$$l_1 = -\frac{\theta_0}{2\beta}. \tag{10}$$

The maximum and minimum reflection coefficients in the Γ_i-plane, Γ_{iM} and Γ_{im}, are readily identified since they lie on the line joining the center of the chart to the center of the data circle. Their counterparts in the Γ_1-plane lie on the real axis and have the same absolute value; i.e., $\Gamma_{1M}=|\Gamma_{iM}|$ and $\Gamma_{1m}=\pm|\Gamma_{im}|$, where the plus sign is taken if Γ_{im} and Γ_{iM} have the same argument; the minus sign, if their arguments differ by π. These values in turn are related to the corresponding values in the Γ_2-plane by the transformer $(n_1:1)$ which must be such that $\Gamma_{2M}=-\Gamma_{2m}$, since the Γ_2 circle has been required to have its center at the origin. One can evidently solve for n_1 in terms of Γ_{1M} and Γ_{1m}, since equations of the form $\Gamma_{1M}=f(n_1^2, \Gamma_{2M})$ and $\Gamma_{1m}=f(n_1^2, -\Gamma_{2M})$ can be written from which Γ_{2M} can be eliminated. Substituting $|\Gamma_{iM}|$ for $|\Gamma_M$ and $\pm|\Gamma_{im}|$ for Γ_{1m}, gives

$$n_1^2 = \sqrt{\frac{(1+|\Gamma_{iM}|)(1\pm|\Gamma_{im}|)}{(1-|\Gamma_{iM}|)(1\mp|\Gamma_{im}|)}}. \tag{11}$$

The representation breaks down, i.e., $n_1^2=\infty$, when $|\Gamma_{iM}|=1$. Here and in (12) the upper sign pertains if the data circle does not include the origin of the Γ_i plane; the lower sign pertains if the circle does include the origin. The value of the reflection coefficient transformer $|\Gamma_\alpha|$ is simply given by $|\Gamma_{2M}|$. Since n_1^2 and Γ_{1M} are now both known, $|\Gamma_{2M}|$ is readily found. One therefore has

$$|\Gamma_\alpha| = \frac{1-\sqrt{\dfrac{(1-|\Gamma_{iM}|)(1\pm|\Gamma_{im}|)}{(1+|\Gamma_{iM}|)(1\mp|\Gamma_{im}|)}}}{1+\sqrt{\dfrac{(1-|\Gamma_{iM}|)(1\pm|\Gamma_{im}|)}{(1+|\Gamma_{iM}|)(1\mp|\Gamma_{im}|)}}}. \tag{12}$$

At this point in the analysis only the two line lengths l and l_2 remain unknown. Of course, $n_2=n_1$ in view of the assumed symmetry. With reference to Fig. 6(d) it is seen that Γ_{3M} and Γ_{3m} are both real and of unity absolute magnitude. By comparing the values of Γ_1, when Γ_3 equals 1 and -1 respectively, it can be shown that $\Gamma_{3M}=1$ and $\Gamma_{3m}=-1$. When a short circuited transmission line, of such length S_M that $\Gamma_3=\Gamma_{3M}=1$, terminates the network at T_2 (see Fig. 4) one may look into the network at z_3 (toward the right) and equate the impedance seen to infinity, since an open circuit is located at this point in view of the special termination. From this relation one obtains

$$\cot\beta l = \frac{1}{n_1^2}\tan\beta(l_2+S_M). \tag{13}$$

Likewise, when the transmission line is of such length S_m that $\Gamma_3=\Gamma_{3m}=-1$, a short circuit appears at z_3 so that one gets

$$-\tan\beta l = \frac{1}{n_1^2}\tan\beta(l_2+S_m). \tag{14}$$

Upon eliminating the $\tan\beta l$ term from (13) and (14), l_2 is found to be

$$l_2 = \frac{1}{2}\left\{\frac{1}{\beta}\cos^{-1}\left[\frac{1+n_1^4}{1-n_1^4}\cos\beta(S_M-S_m)\right] - S_M - S_m\right\}. \tag{15}$$

In view of the arc cosine, (15) does not yield l_2 uniquely. The resulting ambiguity is resolved on the basis of the symmetry requirement; it follows that (d_1-d_2) in Fig. 7 must equal (l_1-l_2) in Fig. 4, i.e.,

$$l_2 = l_1 + d_2 - d_1. \tag{16}$$

Here l_1 is known accurately from (10). As a rule, d_1 and d_2 need not be known accurately, since l_2 need only be found to a first order from (16). That value of l_2 resulting from (15) which most closely corresponds to the value found via (16) is assumed to be correct. It should be noted that such a procedure places greater reliance on the electrical measurement, which is based on relative distance measurements and is consequently generally more accurate, than on absolute distance measurements.

The values of S_M and S_m to be used in (15) may be obtained graphically with reference to Fig. 8. S_M has been so chosen that $\Gamma_3=1$, i.e., in such a manner that the corresponding value in the Γ_i-plane is Γ_{iM}. θ_0, the argument associated with Γ_{iM}, is obtained from the data circle and D_M, the corresponding value of D, can then be computed from (9). S_M is read from a D vs. S curve such as the one shown in Fig. 8(b). In a like fashion, S_m can be abstracted by starting with the argument of Γ_{im}, which in the case shown in Fig. 8 differs from that of Γ_{iM} by π. When Γ_{iM} and Γ_{im} have the same argument (which is the case when the data circle does not include the origin), $D_m=D_M$ and the values of S_m and S_M must be distinguished by considering the approximate values of $|\Gamma_i|$ associated with the two values of S read from the D vs. S curve.

It follows directly from (14) that

$$l = -\frac{1}{\beta}\tan^{-1}\left[\frac{1}{n_1^2}\tan\beta(l_2+S_m)\right], \tag{17}$$

where l_2 may be considered to be known. The two values of the arc tangent leave l in doubt only by a trivial additive term of $\lambda_g/2$.

The Dual Network

The definition of duality employed here is that already given elsewhere.[7] As has been stated, the dual of an n:1 transformer is a 1:n transformer. It can be shown that both transmission lines and reflection coefficient transformers are their own duals, i.e., both the form of the elements and their associated numerical values remain unchanged in going to the dual representation. In consequence the network dual of the modified Wheeler network is identical to the modified Wheeler network itself (see Fig. 4) except for the indicated inversion of the transformers and for a quarter wavelength shift of each reference plane. The dual discussed here is actually the network that results if a single change is made in the original analysis; namely, if instead of defining l_1 so that the center of the circle in the Γ_1-plane falls on the positive real axis, it is so defined that the center falls on the negative real axis.

Relation of the Symmetric Modified Wheeler Network to the Impedance Tee and the Admittance Pi Representations

As pointed out earlier, it is always possible to represent a four-pole at special reference planes T_1' and T_2' by only two transformers joined by a lossy transmission line (see Fig. 5). In particular, a symmetric structure may be represented in this fashion with the further simplification that $n_1 = n_2$. The relationship of this symmetric representation and of its dual to the equivalent impedance Tee and admittance Pi networks at the same reference planes are shown in Fig. 9.

Let the modified Wheeler network at T_1' and T_2' and the corresponding Tee be considered. Upon applying open and short circuit bisection to both networks and equating the real and imaginary parts of the input impedances, one obtains

$$R_{11} + R_{12} = \frac{n^2 \tanh \frac{\alpha l}{2}\left(\tan^2 \frac{\beta l}{2} + 1\right)}{\tan^2 \frac{\beta l}{2} + \tanh^2 \frac{\alpha l}{2}}, \tag{18}$$

$$X_{11} + X_{12} = \frac{n^2 \tan \frac{\beta l}{2}\left(\tanh^2 \frac{\alpha l}{2} - 1\right)}{\tan^2 \frac{\beta l}{2} + \tanh^2 \frac{\alpha l}{2}}, \tag{19}$$

$$R_{11} - R_{12} = \frac{n^2 \tanh \frac{\alpha l}{2}\left(\tan^2 \frac{\beta l}{2} + 1\right)}{1 + \tan^2 \frac{\beta l}{2} \tanh^2 \frac{\alpha l}{2}}, \tag{20}$$

$$X_{11} - X_{12} = -\frac{n^2 \tan \frac{\beta l}{2}\left(\tanh^2 \frac{\alpha l}{2} - 1\right)}{1 + \tan^2 \frac{\beta l}{2} \tanh^2 \frac{\alpha l}{2}}. \tag{21}$$

It is readily shown that in these equations

$$\tanh \frac{\alpha l}{2} = \frac{1 - \sqrt{|\Gamma_\alpha|}}{1 + \sqrt{|\Gamma_\alpha|}}. \tag{22}$$

The parameters of the equivalent Tee are obtained at once from (18) to (22). Since the value of l used here is known only to within an additive constant of $\lambda_g/2$, tan $(\beta l/2)$ may have one of two values depending on the original choice of l. If, say, the quantities in (18) to (21) result from tan $(\beta l/2)$ and the corresponding primed quantities [in (23) below] result from tan $[\beta(l+\lambda_g/2)/2]$ then

$$R_{11} \pm R_{12} = R_{11}' \mp R_{12}', \; X_{11} \pm X_{12} = X_{11}' \mp X_{12}'. \tag{23}$$

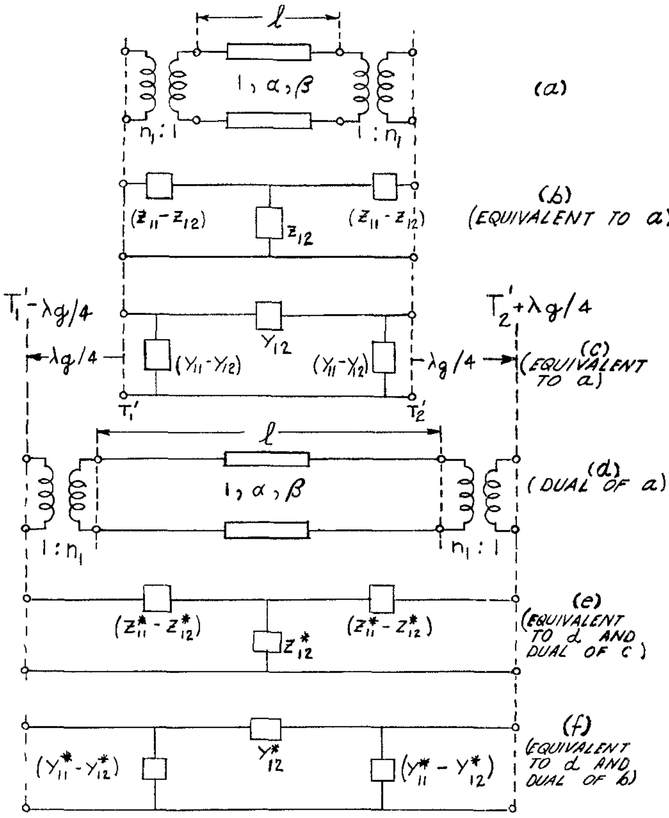

Fig. 9—Modified Wheeler network at special reference planes, its dual and their equivalent Tee and Pi networks.

This ambiguity is identically the one that occurs in other input-output impedance type measurements where the sign of Z_{12}, Y_{12} or S_{12} remains in doubt. Regardless of the choice of sign, the resulting Tee is valid as an impedance transforming network. $\alpha l/2$, on the other hand, is independent of the choice of l in view of the definition of α [see (8)].

[7] H. Kurss, Appendix in L. B. Felsen and A. A. Oliner, "The Precision Measurement of Equivalent Circuit Parameters of Dissipative Microwave Structures," Report R-282-52; PIB-221, Polytechnic Inst. of B'klyn, Microwave Res. Inst.; November, 1952.

Since the planes T_1' and T_2' are specially chosen to eliminate l_1 and l_2, the symmetric modified Wheeler network at these planes is described by only the three parameters n_1, $|\Gamma_\alpha|$ and l (or n_1, α and l). The equivalent Tee, however, requires four parameters for its complete description, i.e., R_{11}, X_{11}, R_{12} and X_{12}. It can consequently be concluded that the choice of the special planes T_1' and T_2' leaves only three of these independent and makes the fourth subject to some restrictive relation. Inspection of (18) to (21) above shows that this relation is

$$\frac{R_{11} + R_{12}}{X_{11} + X_{12}} = -\frac{R_{11} - R_{12}}{X_{11} - X_{12}}, \tag{24}$$

from which it follows that

$$R_{11}X_{11} = R_{12}X_{12}. \tag{25}$$

The parameters of the admittance Pi in terms of the modified Wheeler network can also be obtained via a bisection procedure. If this is done and if the Pi and Tee parameters are compared, the following surprisingly simple relations are found:

$$Y_{11} = Z_{11}/n_1^4, \qquad Y_{12} = Z_{12}/n_1^4. \tag{26}$$

Consequently, the Pi may be obtained from the Tee (or vice versa) almost without effort. Eq. (26) shows that $n_1^4 = |Z|$ where $|Z| \equiv Z_{11}^2 - Z_{12}^2$ (the impedance determinant) and that, consequently, the impedance (or admittance) determinant at T_1' and T_2' is real. Of course, the dual of the Tee at T_1' and T_2' (i.e., the Pi Y_{11}^*, Y_{12}^*) is given by

$$Y_{11}^* = Z_{11}, \qquad Y_{12}^* = Z_{12}, \tag{27}$$

and the dual of the Pi at T_1' and T_2' (i.e., the Tee Z_{11}^*, Z_{12}^*) by

$$Z_{11}^* = Z_{11}/n_1^4, \qquad Z_{12}^* = Z_{12}/n_1^4. \tag{28}$$

The relations yielding the parameters n_1, l and $|\Gamma_\alpha|$ in terms of a given Tee can also be obtained from (18) to (21) by simple but tedious algebraic computations.

Matching the Modified Wheeler Network

Wheeler points out that it is theoretically always possible to achieve a bilateral match for a network of the type shown in Fig. 1. This, of course, also applies to the present network (Fig. 4 or 5). When the input and output transmission lines are of the same characteristic impedance, the match may be achieved here by placing

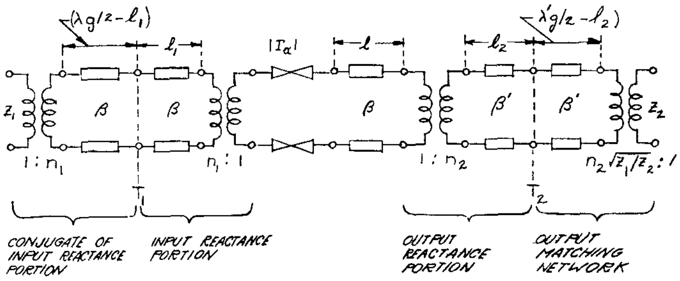

Fig. 10—Modified Wheeler network and adjacent matching structures.

the appropriate conjugate network at each end. When the input and output transmission line characteristic impedances differ, the conjugate of the $1:n_2$ transformer, which is $n_2:1$, must be modified to be $n_2\sqrt{Z_1/Z_2}:1$. The network and the appropriate matching networks (assuming two different characteristic impedances) are shown in Fig. 10. A conjugate for line l may, but need not, be appended since it does not affect the matching of the network.

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